

Simpson or Simpson-like Paradox Examples

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Simpson's paradox refers to the reversal in the direction of an X versus Y relationship when controlling for a third variable Z. Here are some data sets that illustrate the paradox.

Technically the Berkeley admissions data is not a pure Simpson's paradox since reversals do not occur with all graduate departments. Still the effect of controlling for the third variable (graduate department) is substantial, so we might call this a *Simpson-like paradox*. The prenatal example is also Simpson-like.

First, we present a table that summarizes each example. A list of references is at the bottom of the document, and you can consult these for more discussion of each example.

Subject	X	Y	Z	Reference
Berkeley admissions data	sex of applicant	accept or reject	grad program applied to	Freedman, et al 1998, pp. 17-20
Airlines on-time data	airline	on-time or late	airport location	Moore 2003, p. 143
Death sentence data	race of convicted murderer	death sentence: yes or no	Race of murder victim	Witmer 1992, pp. 110-112
Comparing batting averages	person batting	hit or out	year of the at bat	Friedlander 1992, p. 845
Prenatal care	care status	infant mortality	clinic	Bishop, et al 1975, pp. 41-42

Example 1—Berkeley Admissions Data: A study was done by the Graduate Division of the U. of California, Berkeley in the early 70's because a claim of sex bias in graduate admissions had been made. (See *Science*, vol. 187, pages 398-404, 1975, by Bickel, Hammel, and O'Connell.) There were 8442 men who applied for admission and 4321 women. About 44% of the men and 35% of the women were admitted. This appeared to support the claim—namely a bias against women. But by breaking down the data by the graduate department (i.e., lurking variable=graduate department), the researchers (one an associate dean, the other two statisticians) discovered a different story. The table below shows the top 6 departments by enrollments (confidentiality required that names be suppressed), but the pattern observed in these 6 held for the entire graduate program.

Major	MEN		WOMEN	
	No. applicants	Pct. admitted	No. applicants	Pct. admitted
A	825	62	108	82
B	560	63	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

Example 2—Airlines on-time data: Here are numbers of flights on time and delayed for two airlines at five airports in June 1991. The table shows that Alaska Airlines outperforms America West at all 5 cities. If you collapse the table over city, it appears that America West outperforms Alaska.

	Alaska Airlines			America West Airlines		
	On time	Delayed	Delay%	On time	Delayed	Delay%
LA	497	62	11.1%	694	117	14.4%
Phoenix	221	12	5.4%	4840	415	7.9%
San Diego	212	20	8.6%	383	65	14.5%
San Fran.	503	102	16.9%	320	129	28.7%
Seattle	1841	305	14.2%	201	61	23.3%
Total	3274	501	13.3%	6438	787	10.9%

Example 3—Death sentence data: Is the application of the death sentence racially motivated? In 1978, Warren McClesky, a black man, was convicted of killing a white police officer and was sentenced to death in Georgia. In an appeal before the U.S. Supreme Court, McClesky's lawyers argued that imposition of the death penalty in Georgia was racially biased. We look here at data taken from all homicide convictions in the state of Florida between 1976 and 1980 in which the suspect was black or white and the victim was black or white. (Georgia's data are very similar.) The first table shows an apparent bias, but in favor of black suspects. The lurking variable here is the *victim's* race, as the subsequent tables show.

Did convicted person get death sentence?

		YES	NO	Total	(% Yes)
Race of	White	39	308	347	(11.2%)
Suspect	Black	32	345	377	(8.5%)
	Total	71	653	724	(9.8%)

Table for those cases involving white victims:

		Did convicted person get death sentence?			
		YES	NO	Total	(% Yes)
Race of	White	39	279	318	(12.3%)
Suspect	Black	29	121	150	(19.3%)
	Total	68	400	468	(14.5%)

(See next page for black victims.)

Table for those cases involving black victims:

Did convicted person get death sentence?

		YES	NO	Total	(% Yes)
Race of	White	0	29	29	(0.0%)
Suspect	Black	3	224	227	(1.3%)
Total		3	253	256	(1.2%)

Example 4—Comparing batting averages: Here are batting performances for Dave Justice and Andy Van Slyke for two consecutive seasons, 1989 and 1990. Who had the better batting performance for the two years? (Note: At-Bats is the number of attempts to hit the ball for the entire season. Hits is the number of successful attempts. Average=Hits/At-Bats expressed as a decimal to 3 places, so that higher numbers are better.)

Year	Dave Justice			Andy Van Slyke		
	Hits	At-Bats	Average	Hits	At-Bats	Average
1989	12	51	.235	113	476	.237
1990	124	439	.282	140	493	.284
combined	136	490	.278	253	969	.261

Example 5—Prenatal care data:

This example comes from the Harvard School of Public Health. While the original source had been lost, the data were real data. The data are about infant survival in two clinics. Each pregnant mother attended one of two clinics (A or B) and received an amount of prenatal care that was classified as less or more. Does the amount of prenatal care relate to infant survival? It appears that within each clinic, the amount of prenatal care confers little advantage regarding infant survival, but if one aggregates across clinic the picture dramatically changes.

Data from Clinic A:

Amount of Prenatal Care	Infant survival		Total	Survival Rate (%)
	Died	Survived		
Less care	3	176	179	1.7
More care	4	293	297	1.4

Data from Clinic B:

Amount of Prenatal Care	Infant survival		Total	Survival Rate (%)
	Died	Survived		
Less care	17	197	214	7.9
More care	2	23	25	8.0

References

Bishop, Yvonne M., Fienberg, Stephen E., and Holland, Paul W. (1975), *Discrete Multivariate Analysis: Theory and Practice*, Cambridge, Massachusetts: The MIT Press.

Freedman, David, Pisani, Robert, and Purves, Roger (1998), *Statistics* (3rd ed.), New York, NY: W.W. Norton and Company.

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Moore, David S. (2003), *The Basic Practice of Statistics* (3rd ed.), New York, NY: W.H. Freeman.

Witmer, Jeffrey A. (1992), *Data Analysis: An Introduction*, Engelwood Cliffs, NJ: Prentice-Hall.