Homework 5: Due Wednesday, September 8

**Problem 1:** Let \( f_n \) be the \( n^{th} \) Fibonacci number as defined in Problem 2 on Homework 3. Show that \( \gcd(f_n, f_{n+1}) = 1 \) for all \( n \in \mathbb{N}^+ \).

**Problem 2:** Let \( a, b, c \in \mathbb{Z} \) with \( a > 0 \). Show that \( \gcd(ab, ac) = a \cdot \gcd(b, c) \).

**Problem 3:** Let \( a, b, c \in \mathbb{Z} \). Show that the following are equivalent:
- \( \gcd(ab, c) = 1 \)
- \( \gcd(a, c) = 1 \) and \( \gcd(b, c) = 1 \)

**Problem 4:** Let \( a, b \in \mathbb{N}^+ \) and let \( d = \gcd(a, b) \). Since \( d \) is a common divisor of \( a \) and \( b \), we may fix \( k, \ell \in \mathbb{N} \) with \( a = kd \) and \( b = \ell d \). Let \( m = k \ell d \).
  a. Show that \( a \mid m, b \mid m, \) and \( dm = ab \).
  b. Show that \( \gcd(k, \ell) = 1 \).
  c. Suppose that \( n \in \mathbb{Z} \) is such that \( a \mid n \) and \( b \mid n \). Show that \( m \mid n \).

Because of parts a and c above, the number \( m \) is called the least common multiple of \( a \) and \( b \) and is written as \( \operatorname{lcm}(a, b) \). Since \( dm = ab \) from part a, it follows that \( \gcd(a, b) \cdot \operatorname{lcm}(a, b) = ab \).