Homework 20 : Due Friday, October 29

**Problem 1:** Let $GL_n(\mathbb{R})$ act on $\mathbb{R}^n$ as usual, so $A \cdot x = Ax$.

a. Find (with proof) the orbits of the action of the subgroup $SL_2(\mathbb{R})$ on $\mathbb{R}^2$.

b. Find (with proof) the orbits of the action of $GL_n(\mathbb{R})$ on $\mathbb{R}^n$.

*Hint:* For part b especially, it is possible to build lots of matrices to make things work, but you can by with far less effort if you use some theory.

**Problem 2:** Suppose that $G$ acts on $X$. We saw in class that for each $a \in G$, the function $\pi_a : X \to X$ defined by $\pi_a(x) = a \cdot x$ is a permutation of $X$. Define $\varphi : G \to S_X$ by letting $\varphi(a) = \pi_a$. Show that $\varphi$ is a homomorphism.

**Problem 3:** Suppose that $G$ acts on $X$. Let $H = \{a \in G : a \cdot x = x \text{ for all } x \in X\}$. Show that $H$ is a normal subgroup of $G$. $H$ is called the kernel of the action. Notice that $H$ is in the intersection of all the stabilizers $G_x$.

**Problem 4:** Let $G = \mathbb{R}$ (under addition) and let $X = \mathbb{R}^2$. Define a function from $G \times X$ to $X$ by $a \cdot (x, y) = (x + ay, y)$.

a. Show that $\cdot$ is a action of $G$ on $X$.

b. Describe the orbits of the action geometrically. Be careful!

c. Describe the stabilizers of each point.

**Problem 5:** Let $G = S_3$ and let

$$X = \{(1, 2, 3) \times \{1, 2, 3\} = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Define a function from $G \times X$ to $X$ by $\sigma \cdot (x, y) = (\sigma(x), \sigma(y))$.

a. Show that $\cdot$ is a action of $G$ on $X$.

b. Find the orbits and stabilizers of each point.