Problem 1: Let $G$ be a group. Suppose that $H$ and $K$ are finite subgroups of $G$ such that $|H|$ and $|K|$ are relatively prime. Show that $H \cap K = \{e\}$.

Problem 2: Consider $\mathbb{Q}$ as group under the operation of addition. Notice that $\mathbb{Z}$ is a subgroup of $\mathbb{Q}$. Show that $[\mathbb{Q} : \mathbb{Z}] = \infty$.

Problem 3:
(a) Let $G$ be a group with the property that $a^2 = e$ for all $a \in G$. Show that $G$ is abelian.
(b) Show that every group of order 4 is abelian.

Note: Recall that $U(\mathbb{Z}/8\mathbb{Z})$ is an example of abelian group of order 4 which is not cyclic. Since 2, 3, and 5 are prime, it follows that the smallest possible order of a nonabelian group is 6. Indeed, $S_3$ is an example of such a group.

Problem 4: Let $p, k \in \mathbb{N}^+$ with $p$ prime. Suppose that $G$ is a group with $|G| = p^k$. Show that $G$ has an element of order $p$.

Problem 5: Let $G$ be a group. Suppose that $H$ and $K$ are both subgroups of $G$. Define a relation on $G$ by letting $a \sim b$ mean that there exists $h \in H$ and $k \in K$ with $b = hak$.
(a) Show that $\sim$ is an equivalence relation on $G$.
(b) The equivalence classes of $\sim$ are called double cosets. Find the double cosets in the case where $G = A_4$ and $H = K = \langle (1 \ 2 \ 3) \rangle$. 