Problem 1: Let \( \times \) be the cross product on \( \mathbb{R}^3 \).

a. Is \( \times \) an associative operation on \( \mathbb{R}^3 \)? Explain.

b. Does \( \times \) have an identity on \( \mathbb{R}^3 \)? Explain.

Problem 2: Consider the set \( \mathbb{R}_{\geq 0} = \{ x \in \mathbb{R} : x \geq 0 \} \) of nonnegative reals. Let \( * \) be the binary operation on \( \mathbb{R}_{\geq 0} \) given by exponentiation, i.e. \( a * b = a^b \).

a. Is \( * \) an associative operation on \( \mathbb{R}_{\geq 0} \)? Explain.

b. Does \( * \) have an identity on \( \mathbb{R}_{\geq 0} \)? Explain.

Problem 3: Let \( (G, \cdot, e) \) be a group and let \( a, b, c, d \in G \). Show that

\[(a \cdot b) \cdot (c \cdot d) = (a \cdot (b \cdot c)) \cdot d\]

Problem 4: Let \( G \) be the set of all \( 3 \times 3 \) matrices of the form

\[
\begin{pmatrix}
1 & x & y \\
0 & 1 & z \\
0 & 0 & 1
\end{pmatrix}
\]

where \( x, y, z \in \mathbb{R} \). Now we know from linear algebra that matrix multiplication is associative and that the identity matrix is

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

so the identity matrix is in \( G \).

a. Show that if \( A, B \in G \), then \( AB \in G \). Thus, matrix multiplication is indeed a binary operation on \( G \).

b. Show that if \( A \in G \), then \( A \) is an invertible matrix and moreover \( A^{-1} \in G \). Thus, \( G \) is a group under matrix multiplication. (Notice that if some matrix \( A \in G \) was invertible as a matrix but its inverse was not in \( G \), then \( G \) would not be a group.)