Problem Set 8

**Problem 1:** How many zeros does 10000! end with?

**Problem 2:** Let \( R \) be the region consisting of the points \((x, y)\) of the cartesian plane satisfying both \(|x| - |y| \leq 1\) and \(|y| \leq 1\). Sketch the region \( R \) and find its area.

**Problem 3:** Let \( n \in \mathbb{Z} \). Show that \( \gcd(n^2+1, (n+1)^2+1) \) is either 1 or 5.

*Problem 4:* Find all positive integers \( n \) such that \( n = d(n)^2 \), where \( d(n) \) equals the number of positive divisors of \( n \) (for example, \( d(9) = 3 \)).

*Problem 5:* Prove that the expression

\[
\frac{\gcd(m, n)}{n} \binom{n}{m}
\]

is an integer whenever \( n \geq m \geq 1 \).

*Problem 6:* Given \( n \in \mathbb{N}^+ \), let \( [n] = \{1, 2, 3, \ldots, n\} \).

a. For which values of \( n \) is it possible to express \( [n] \) as the union of two non-empty disjoint subsets so that the elements in the two subsets have equal sum?

b. For which values of \( n \) is it possible to express \( [n] \) as the union of three non-empty disjoint subsets so that the elements in the three subsets have equal sum?

*Problem 7:* Let \( d \) be a real number. For each integer \( m \geq 0 \), define a sequence \( \{a_m(j)\} \) by the condition

\[
a_m(0) = \frac{d}{2^m} \quad a_m(j + 1) = (a_m(j))^2 + 2a_m(j)
\]

Evaluate \( \lim_{n \to \infty} a_n(n) \).