Written Assignment 9 : Due Wednesday, April 27

Problem 1: Find values of $c$ and $d$ such that the matrix

\[
\begin{bmatrix}
3 & 1 \\
c & d
\end{bmatrix}
\]

has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.

Problem 2: Let $A$ be an $n \times n$ idempotent matrix.

a. Show that $\text{Null}(A) \cap \text{Col}(A) = \{0\}$.

b. Show that for every $v \in \mathbb{R}^n$, there exists $u \in \text{Null}(A)$ and $w \in \text{Col}(A)$ with $v = u + w$.

Problem 3: Define a sequence of numbers as follows. Let $g_0 = 0$, $g_1 = 1$, and $g_n = \frac{1}{2}(g_{n-1} + g_{n-2})$ for $n \geq 2$. In other words, the $n^{th}$ term of the sequence is the average of the two previous terms.

a. Find a general equation for $g_n$.

b. As $n$ gets large, the values of $g_n$ approach a fixed number. Find that number.

c. Suppose that you change the initial starting values of $g_0$ and $g_1$. As $n$ gets large, must the terms of the sequence still approach a fixed number? If so, explain why and determine that number in terms of $g_0$ and $g_1$. If not, find an example where the terms of sequence do not approach one fixed number.