Written Assignment 3 : Due Wednesday, February 16

Problem 1: Suppose that \( \{v_1, v_2, \ldots, v_n\} \) is a linearly independent set of vectors in \( \mathbb{R}^n \) (notice the same \( n \)). Explain why \( \text{Span}\{v_1, v_2, \ldots, v_n\} = \mathbb{R}^n \).

Problem 2: Suppose that \( \{v_1, v_2, \ldots, v_k\} \) is a linearly independent set of vectors in \( \mathbb{R}^n \). Suppose that \( c_i \) and \( d_i \) are scalars such that:

\[
    c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = d_1 v_1 + d_2 v_2 + \ldots d_k v_k
\]

Show that \( c_i = d_i \) for all \( i \).

Problem 3: Suppose that \( T: \mathbb{R}^n \rightarrow \mathbb{R}^m \) is a linear transformation. Suppose that \( u \in \text{Span}\{v_1, v_2, \ldots, v_k\} \). Show that \( T(u) \in \text{Span}\{T(v_1), T(v_2), \ldots, T(v_k)\} \).