Problem 1: Consider the function $f: \mathbb{Z} \to \mathbb{Z}^2$ given by $f(n) = (3n^2 - 77, 5n + 6)$.

a. Is $f$ injective? Justify your answer carefully.

b. Is $f$ surjective? Justify your answer carefully.

Problem 2: Suppose that $A$, $B$, and $C$ are sets and that both $f: A \to B$ and $g: B \to C$ are surjective functions. Show that the function $g \circ f: A \to C$ is surjective.

Problem 3: Suppose that we have a function $f: \mathbb{R} \to \mathbb{R}$ with the property that $f(x \cdot y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. Suppose that $f(2) = 5$ and $f(3) = 7$. What is $f(\frac{1}{2})$? Explain.

Hint: What can you say about $f(1)$?