Problem Set 17: Due Friday, November 14

Problem 1: Does

\[ \text{Span} \left( \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \mathbb{R}^3 ? \]

Explain.

Problem 2: Given \( b_1, b_2, b_3 \in \mathbb{R} \), determine necessary and sufficient conditions so that

\[ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \text{Span} \left( \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right) \]

is true.

Problem 3: Working in \( \mathcal{P}_3 \), consider the following functions:

- \( f_1(x) = x^3 + 2x^2 + x \).
- \( f_2(x) = -3x^3 - 5x^2 + x + 2 \).
- \( f_3(x) = x^2 - x + 1 \).
- \( g(x) = x^3 + 8x^2 + 7 \).

Is \( g \in \text{Span}(f_1, f_2, f_3) ? \) Explain.

Problem 4: Let \( V \) be the vector space of all \( 2 \times 2 \) matrices. Does

\[ \text{Span} \left( \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 7 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix} \right) = V ? \]

Explain.

Problem 5: Show that the only subspaces of \( \mathbb{R} \) are \( \{0\} \) and \( \mathbb{R} \).

Hint: Suppose that \( W \) is a subspace of \( \mathbb{R} \) with \( W \neq \{0\} \). Explain why every element of \( \mathbb{R} \) is in \( W \).

Problem 6: In Problem 5 on Problem Set 14, you showed that

\[ W = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0 \right\} \]

was a subspace of \( \mathbb{R}^3 \). Show that

\[ W = \text{Span} \left( \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \]

by giving a double containment proof.

Aside: Using this result, we can instead apply Proposition 3.16 to conclude that \( W \) is a subspace of \( \mathbb{R}^3 \).