Problem Set 12: Due Friday, October 17

**Problem 1:** Find the eigenvalues of the matrix
\[
\begin{pmatrix}
5 & -1 \\
-7 & 3
\end{pmatrix}
\]

**Problem 2:** Find the eigenvalues of the matrix
\[
\begin{pmatrix}
1 & 8 \\
2 & 1
\end{pmatrix}
\]
and then find (at least) one eigenvector for each eigenvalue.

**Problem 3:** Find the eigenvalues of the matrix
\[
\begin{pmatrix}
2 & -1 \\
1 & 4
\end{pmatrix}
\]
and then find (at least) one eigenvector for each eigenvalue.

**Problem 4** Find values for \(c\) and \(d\) such that the matrix
\[
\begin{pmatrix}
3 & 1 \\
c & d
\end{pmatrix}
\]
has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.

**Problem 5:** Explain why a \(2 \times 2\) matrix \(A\) is invertible if and only if 0 is not an eigenvalue of \(A\).