Problem Set 11

IMMERSE 2005 Analysis Course

1. Complete #2 from Problem Set 8, if you haven't finished it already.

2. Prove Claim 2 from lecture: Given $\epsilon > 0$ and h_1 a continuous real-valued function on $\mathbb{R}$ with compact support, there exists a step function h_2:

$$h_2(x) = \sum_{i=1}^{n} \alpha_i \chi_{[a_i,b_i]}(x)$$

   such that $\|h_1 - h_2\|_2 < \epsilon$. (Recall that the characteristic function χ is defined as follows. Let $E \subseteq \mathbb{R}$. Then $\chi_E(x) = 1$ for $x \in E$ and $\chi_E(x) = 0$ for $x \notin E$.)

3. Let ψ be the Haar wavelet function $\psi(x) = \chi_{[0,\frac{1}{2})}(x) - \chi_{[\frac{1}{2},1)}(x)$, and denote $\psi_{jk}(x) = 2^{\frac{j}{2}}\psi(2^j x - k)$, where $j, k \in \mathbb{Z}$.

   Let

$$h(x) = \begin{cases} 1 & 0 \le x < \frac{1}{4} \\ 3 & \frac{1}{4} \le x < \frac{1}{2} \\ -1 & \frac{1}{2} \le x < \frac{3}{4} \\ 1 & \frac{3}{4} \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find $g = \sum_{j=1}^{n} \sum_{k=1}^{m} c_{jk}\psi_{jk}$ for which $\|h - g\|_2 \le \frac{1}{2}$.