## Review Problems for Exam 2

On the second exam, the following formula will be on the front page:

$$\left(\frac{dx}{dt}\right)^2 \frac{d^2y}{dx^2} + \left(\frac{d^2x}{dt^2} + p(t)\frac{dx}{dt}\right)\frac{dy}{dx} + q(t)y = 0.$$

1. (3.1 # 17) Find a differential equation whose general solution is $y = c_1e^{2t} + c_2e^{-3t}$.
2. (2.9 # 44) Find the solution of the differential equation $y'' + y(y')^3 = 0$.
3. (2.9 # 39) Find the solution of the differential equation $2t^2y'' + (y')^3 = 2ty', t > 0$.
4. (3.4 # 16) Find the general solution of $y'' + 4y' + 6.25y = 0$.
5. (3.4 # 37) Solve $ty'' + (t^2 - 1)y' + t^3y = 0$.
6. (3.5 # 19) If the roots of the characteristic equation are real, show that a solution of $ay'' + by' + cy = 0$ can take on the value zero at most once.
7. (3.5 # 28) Use reduction of order to find a second solution:
$$(x - 1)y'' - xy' + y = 0, x > 1; y_1(x) = e^x.$$
8. (3.6 # 6) Find the general solution of $y'' + 2y + y = 2e^{-t}$.
9. (3.6 # 32) Show that if $a$, $b$, and $c$ are all positive, then every solution of the differential equation $ay'' + by' + cy = d$ appoaches $d/c$ as $t \to \infty$. What if $c = 0$?
10. (3.8 # 10) A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in./sec, find its position $u$ at any time $t$.
11. (3.8 # 14) Show that the period of motion of an undamped vibration of a mass hanging from a vertical spring is $2\pi\sqrt{L/g}$, where $L$ is the elongation of the spring due to the mass and $g$ is the acceleration due to gravity.
12. (3.9 # 4) Write the expression as a product of two trigonometric functions of different frequencies: $f(t) = \sin 3t + \sin 4t$.
13. (3.9 # 12) A spring-mass system has a spring constant of 3 N/m. A mass of 2 kg is attached to the spring and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity. If the system is driven by an external force of $3\cos 3t - 2\sin 3t$ N, determine the steady-state response. Express your answer in the form $R\cos(\omega t - \delta)$.

14. (7.1 # 15) Consider the linear homogeneous system

$$x' = p_{11}(t)x + p_{12}(t)y,$$

$$y' = p_{21}(t)x + p_{22}(t)y.$$

Show that if $x = x_1(t), y = y_1(t)$ and $x = x_2(t), y = y_2(t)$ are two solutions of the given system, then $x = c_1x_1(t) + c_2x_2(t), y = c_1y_1(t) + c_2y_2(t)$ is also a solution for any constants $c_1$ and $c_2$.

15. (7.1 # 2) Transform the equation into a system of first order equations.

$$u'' + .5u' + 2u = 3\sin t$$

16. (7.1 # 10) Transform the given system into a single equation of second order. Then find $x_1$ and $x_2$ that also satisfy the given initial conditions.

$$x_1' = x_1 - 2x_2, x_1(0) = -1$$

$$x_2' = 3x_1 - 4x_2, x_2(0) = 2$$

17. (7.5 # 4) Find the general solution of the equation

$$x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x$$

Use the eigenvectors and eigenvalues to draw a sketch of the phase portrait. Describe the origin (e.g. stable node, unstable spiral, etc.)

18. (7.5 # 16) Solve the initial value problem and describe the behavior of the solution as $t \to \infty$.

$$x' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} x, x(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Describe the origin (e.g. stable node, unstable spiral, etc.)

19. (7.6 # 6) Find the general solution of

$$x' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} x$$

Sketch a phase portrait. Describe the origin.

20. (7.6 # 14) Determine the eigenvalues is terms of $\alpha$ and find the critical value or values of $\alpha$ where the qualitative nature of the phase portrait for the system changes.

$$x' = \begin{pmatrix} 0 & -5 \\ 1 & \alpha \end{pmatrix} x$$